Gaussian Process Kernel Crossover for Automated Forex Trading System

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***Abstract*— Due to time varying volatility of forex financial time series price data, the traditional technical analysis such as simple moving average (SMA) and exponential moving average (EMA) cannot capture the dynamic time varying in the market. The combination of Gaussian Process kernel framework and classical trendline crossover strategy is proposed with new objective function for optimization trading performance. The experimental results demonstrate better trading performance for both price prediction and accumulated profit return.**

Keywords *: **Gaussian Process Kernel, Forex, Automated trading***

I. Introduction

The foreign exchange market (Forex or FX) is the largest financial market with average trading volume of more than $5.1 trillion per day which is more than the total daily stock market trading volume in the world. Traders are composed of individual investors, governments, central banks, commercial banks, and financial institutions. The currencies are traded against one another in pairs. FX market does not set a currency's value but rather determines its relative value by setting price of one currency which paid for another. Ex: 1 USD is worth 1.0714 EUR or 1.2801 GBP. According to the 2016, the most heavily traded currency pairs were EURUSD (23.0%), USDJPY (17.7%) GBPUSD (9.2%) [1]. EURUSD is the most popular trading currency pairs. Therefore, the currency pairs EURUSD is used for experimental data in this paper.

The problem of analysis price trends in the FX market is an important task to forecast market direction. Technical analysis indicators are considered for more useful tools in forecasting trend or trend following trading strategy. The common trend forecasting indicators for generating trading signal by trend line golden cross strategy are composed of double trend line such as double simple moving average (SMA) or double exponential moving average (EMA) crossovers, calculated from two moving average with difference time length as descried in section II.

From this point, the model in financial time series to forecast the price trend is like models in signal processing applications from the electrical engineering section. For example, reduced noise in the price is filtered by Kalman filter and Particle filter [3]. With the recently development forecasting signal from electrical engineering applications [4,5], the authors present the combination of financial trend line golden cross strategy and two lead-lag Gaussian Process kernel [6] with new objective function for trend following trading strategy. These trading signal are based on the concept of two lead-lag golden crossover time series Gaussian Process kernel.

II. Lead-Lag Trendline Crossover Overview

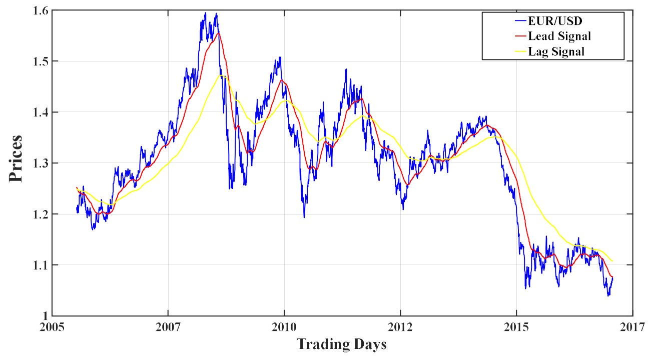

Figure 1. EMA lead-lag trend line with $\theta_{lead} = 50$, $\theta_{lag} = 200$

An original method for lead-lag trend line signal cross-over develops simple trading systems which base on two either simple moving average (SMA) or exponential moving average (EMA) cross-overs of different parameter length θ (usually faster $\theta_{lead} = 5$, slower $\theta_{lag} = 35$ days for short term decision and faster $\theta_{lead} = 50$, slower $\theta_{lag} = 200$ for medium term decision). The computations of SMA and EMA operating on sampling price EUR/USD_t ($t = 1,2,3\ldots N$) during period θ are [1, 7, 8]

$$SMA_t(\theta) = \frac{\sum_{k=t-\theta+1}^{t} \text{EUR/USD}_k}{\theta} \tag{1}$$

$$EMA_t(\theta) = \alpha \text{EUR/USD}_t + (1-\alpha) EMA_{t-1}(\theta) \tag{2}$$

Where subscript $t = 1,2,3\ldots N$ denote the sampling time and $\alpha = 2/(\theta+1)$. The lead $lead_t(\theta_{lead})$ and lag $lag_t(\theta_{lag})$ signal is computed by selection $\theta_{lead} < \theta_{lag}$ on either $SMA_t(\theta)$ or $EMA_t(\theta)$ method as shown in an example in Fig.1. A trading system using two trend lines cross-over would give a trading signal by invoking the golden cross strategy. Buy (long) trading signal when the lead (faster) trend line is rising and cross above the lag (slower) trend line. A sell (short) trading signal would be given

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when the lead trend line crosses below the lag trend line. Denote that from observation sampling set $\{\text{EUR/USD}_t\}_{t=1}^{N}$, there are M trading orders in system ($M << N$). The j^{th} trading order ($j \in M$) for golden cross strategy trading signal at sampling time t is define by

$$Open\ Order_j = \begin{cases} Buy, & if \quad lead_t > lag_t \\ & and \quad lead_{t-1} < lag_{t-1} \\ Sell, & if \quad lead_t < lag_t \\ & and \quad lead_{t-1} > lag_{t-1} \end{cases} \tag{3}$$

If none of conditions match, the previous sampling trading strategy (buy or sell) is held status. Moreover, the j^{th} trading order's (take profit or stop loss) at time t^* ($t^* > t$) is also accomplished by golden cross strategy

$$Close\ Order_j = \begin{cases} Buy, & if \quad lead_{t^*} < lag_{t^*} \\ & and \quad lead_{t^*-1} > lag_{t^*-1} \\ Sell, & if \quad lead_{t^*} > lag_{t^*} \\ & and \quad lead_{t^*-1} < lag_{t^*-1} \end{cases} \tag{4}$$

From Equation. (3)-(4), The j^{th} golden cross strategy profit denoted by r_j, which start open trading order at time t and close trading order at time t^* with ($t^* > t$), is computed by

$$r_j = \begin{cases} \text{EUR/USD}_{t^*} - \text{EUR/USD}_t & , \quad Buy \\ -\left(\text{EUR/USD}_{t^*} - \text{EUR/USD}_t\right) & , \quad Sell \end{cases} \tag{5}$$

The growing of cumulative profit $\sum_{i=1}^{M} r_i$ in the system and the number of trading orders M will depend on the characteristic of lead-lag trend line signal. However, the low complexity trend line SMA and EMA will also generate more false trading signals if θ_{lead}, θ_{lag} are not appropriate selections and sideway price movement due to high volatility. To reduce false trading signal and increase capability trading performance, Gaussian process kernel learning is presented for replacing SMA and EMA method for lead-lag smart curve learning trend line.

III. The Concept of Objective Function for Lead-Lag Crossover Investment Strategy

As mentioned in section II, the objective function goal in the optimization competition is to obtain the best parameters θ_{lead}, θ_{lag} for a trading lead-lag crossover strategy with optimal low risk and continuous profit performance as measured by the Sharpe ratio strategy [9]. The Sharpe ratio at equation (6) is determined by the mean average of the returns of strategy with the standard deviation of those returns. Thus, a lower volatility and continuous growing up of returns will lead to a greater Sharpe ratio. Moreover, Sharpe ratio is defined by

$$S = \frac{E\{r_i\}}{\sigma_r} \tag{6}$$

Where mean $E\{r_i\}$ and standard deviation σ_r are calculated by $E\{r_i\} = \sum_{i=1}^{M} r_i \Big/ M$, $\sigma_r = \left(\sum_{i=1}^{M} \sqrt{\left(r_i - E\{r_i\}\right)^2} \right) \Big/ M$. In order to complete the Sharpe ratio's objective function from equation (6) ,The $lead_t(\theta_{lead})$ and $lag_t(\theta_{lag})$ trading signal are compute by maximization Sharpe ratio to θ_{lead}, θ_{lag} by equation (7)

$$\begin{aligned} \{lead_t, lag_t\} &= \underset{\theta_{lead}, \theta_{lag}}{\arg\max} \{S\} \\ &= \underset{\theta_{lead}, \theta_{lag}}{\arg\max} \left\{ \frac{\sum_{i=1}^{M} r_i}{\sum_{i=1}^{M} \sqrt{\left(r_i - E\{r_i\}\right)^2}} \right\} \end{aligned} \tag{7}$$

The examples of SMA and EMA for selected optimal parameters over the maximum Sharpe ratio, which are corresponding to variables θ_{lead} and θ_{lag} by using the 10 years daily of observation data $\{\text{EUR/USD}_t\}_{t=1}^{N}$ from 15 January 2007 to 30 January 2017 are shown in Figure 1. and Figure 2. The searching for optimal parameters (θ_{lead}, θ_{lag}) is to compute the Sharp ratio for each pair of $1 \le \theta_{lead} \le 200$, $1 \le \theta_{lag} \le 200$ and select the best (θ_{lead}, θ_{lag}) matching with maximum Sharp ratio. Thus, the maximum Sharp ratio for EMA method are 0.0463 at the best $\theta_{lead} = 5$, $\theta_{lag} = 31$.The maximum Sharp ratio for SMA method are 0.0352 at the best $\theta_{lead} = 60$, $\theta_{lag} = 101$.

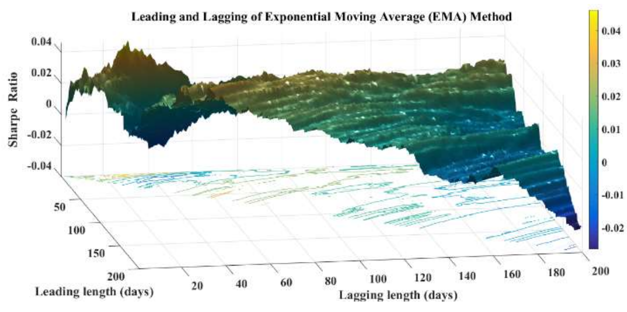

Figure 1. Sharp ratio of EMA method $1 \le \theta_{lead} \le 200$, $1 \le \theta_{lag} \le 200$

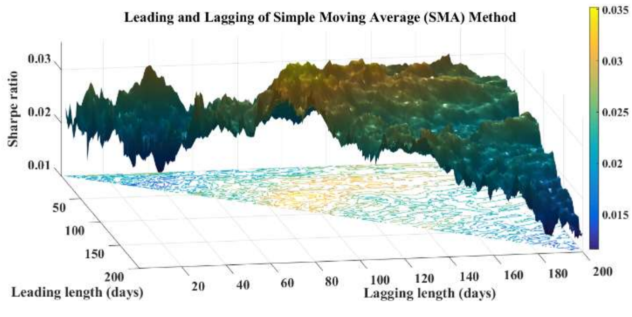

Figure 2. Sharp ratio of SMA method $1 \le \theta_{lead} \le 200$, $1 \le \theta_{lag} \le 200$

IV. PROPOSED LEAD-LAG GAUSSIAN PROCESS KERNEL

In this section, we design lead-lag Gaussian process kernel signal for smart trend line crossover trading algorithm as described in section II. The Gaussian process [6] is defined as a probability density function for multivariate Gaussian distribution over function. Any subset of observation data $\mathbf{y}=\left[\text{EUR/USD}_t\right]_1^N$ from the Gaussian process follows the joint multivariate Gaussian distribution between sampling data $\mathbf{y}$ and trend line $GP_t(\theta)$ where θ is a set of hyperparameters. Therefore, the lead signal $lead_t = GP_t(\theta_{lead})$ and lag signal $lag_t = GP_t(\theta_{lag})$ of Gaussian process trend line are determined by the mean $m(t)$ and covariance (kernel) function $k(t,t'\mid\theta)=k(\tau\mid\theta)$ where $\tau = t-t'$

$$lead_t = GP_t(\theta_{lead}) = \mathcal{N}\left(m_{lead}(t), k(\tau_t \mid \theta_{lead})\right) \tag{8}$$

$$lag_t = GP_t(\theta_{lag}) = \mathcal{N}\left(m_{lag}(t), k(\tau_t \mid \theta_{lag})\right) \tag{9}$$

The integration of kernel functions $k(\tau\mid\theta)$ in Gaussian process further enhances its flexibility in adapting trend line function $GP_t(\theta)$ to the same data sets $\mathbf{y}=\left[\text{EUR/USD}_t\right]_1^N$. In fact, Gaussian processes can be considered as the limit of neural networks with infinite hidden units [[6.Finally, the mathematics closed form for trend line $GP_t(\theta)$ is calculated by [4-6]

$$GP_t(\theta) = m(t) = \mathbf{k}^T(\theta)\left(\mathbf{K}(\theta)+\sigma^2 I_N\right)^{-1}\mathbf{y} \tag{10}$$

where

$$\mathbf{K}(\theta) \triangleq \begin{bmatrix} k(\tau_0\mid\theta) & k(\tau_{-1}\mid\theta) & \dots & k(\tau_{-(N-1)}\mid\theta) \\ k(\tau_1\mid\theta) & k(\tau_0\mid\theta) & \dots & k(\tau_{-(N-2)}\mid\theta) \\ \vdots & \vdots & \vdots & \vdots \\ k(\tau_{N-1}\mid\theta) & k(\tau_{L-2}\mid\theta) & \dots & k(\tau_0\mid\theta) \end{bmatrix}_{N\times N} \tag{11}$$

and

$$\mathbf{k}(\theta) \triangleq \left[k(\tau_1\mid\theta) \quad \dots \quad k(\tau_N\mid\theta)\right]^T \tag{12}$$

The $k(\tau\mid\theta)$ is selected by popular squared exponential (SE) kernel function [4, 6] with hyper parameters $\theta=\{\sigma^2,\alpha\}$.

$$k_{SE}(\tau\mid\theta) = \sigma^2 \exp\left(-\alpha\|\tau\|^2\right) \\ \theta = \{\sigma^2,\alpha\} \tag{13}$$

The characteristic of trend line signal $GP_t(\theta)$, generated by SE kernel $k(\tau\mid\theta)$, is adaptive smooth line. The magnitude and smoothing factor are controlled by hyperparameters σ^2 and α.

V. PROPOSED OBJECTIVE FUNCTION OF LEAD-LAG GAUSSIAN PROCESS KERNEL

In order to perform trading algorithm based on combination of lead-lag Gaussian process kernel in section IV and maximization of Sharp ratio objective function at eq. (6) in section III, the authors design a new objective function $f(\theta)$ for finding minimization function of four input hyper parameters θ by

$$\theta = \{\theta_{lead},\theta_{lag}\} \\ = \{\sigma^2_{lead},\alpha_{lead},\sigma^2_{lag},\alpha_{lag}\} \tag{14}$$

As shown in figure 1. and figure 2. The characteristics of objective function $f(\theta)$ similarly contain multiple maxima or minima. the minimization of $f(\theta)$ can easy to use global optimization toolbox with Genetic algorithm in MATLAB program to solve these optimization with black-box functions where the hidden constraint functions in $f(\theta)$ are composed of $GP_t(\theta_{lead})$ and $GP_t(\theta_{lag})$ with kernel functions $k_{SE}(\tau\mid\theta_{lead})$ and $k_{SE}(\tau\mid\theta_{lag})$. Generally, the searching minimization algorithm are necessary to employ good initial parameter $\theta^{(0)}$ due to multiple maxima or minima. The reasonable $\theta^{(0)}$ make a stability to find global solution of trading strategy. Finally, $f(\theta)$ is defined in table 1.

TABLE 1. OBJECTIVE FUNCTION $f(\theta)$

Objective Function	$f(\theta)$
Get observation data	$\mathbf{y}=\left[\text{EUR/USD}_t\right]_1^L$
Compute kernel function	$k_{SE}(\tau\mid\theta_{lead})$ and $k_{SE}(\tau\mid\theta_{lag})$
Compute Gaussian Process kernel lead-lag trend line	
	$lead_t = GP_t(\theta_{lead}) = \mathbf{k}^T(\theta_{lead})\left(\mathbf{K}(\theta_{lead})+\sigma^2 I_N\right)^{-1}\mathbf{y}$
	$lag_t = GP_t(\theta_{lag}) = \mathbf{k}^T(\theta_{lag})\left(\mathbf{K}(\theta_{lag})+\sigma^2 I_N\right)^{-1}\mathbf{y}$
Calculate return r_i for $i=1,2,\dots M$ by cross-over strategy	
Compute Shape ratio	$S(\theta)=\dfrac{E\{r_i\}}{\sigma_r}$
Return	$f(\theta) = -S(\theta)$

VI. EXPERIMENTS AND RESULTS

To verify the efficiency in making the profits between the proposed lead-lag Gaussian Process kernel algorithm and the comparative study of lead-lag SMA and EMA indicator method, tests are conducted and results are presented in this section. The currency Euro and Dollar (EUR/USD) for the period 15 January

2007 to 30 January 2017 are selected with the daily value of Open, High, Low, and Close (OHLC). For comparative study performance, the lead-lag EMA and lead-lag SMA with highest Sharp ratio 0.0463 and 0.0352 with $\theta_{lead} = 5$, $\theta_{lag} = 31$ for EMA method and $\theta_{lead} = 60$, $\theta_{lag} = 101$ for SMA method as descried in section III are analyzed together lead-lag Gaussian Process kernel in Figure 3-5. The optimal hyper parameters $\theta = \{\theta_{lead}, \theta_{lag}\}$ in Gaussian Process kernel are computed by black-box Genetic algorithm in Table 1. where the optimal hyper parameters are $\theta_{lead} = \{33.7645 \quad 6.51\}$ and $\theta_{lag} = \{14.312 \quad 2.875\}$ with the highest Sharp ratio 0.0821. Furthermore, the characteristic of lead-lag trend line and the cumulative profit for Gaussian Process kernel are present in Figure 4-5.

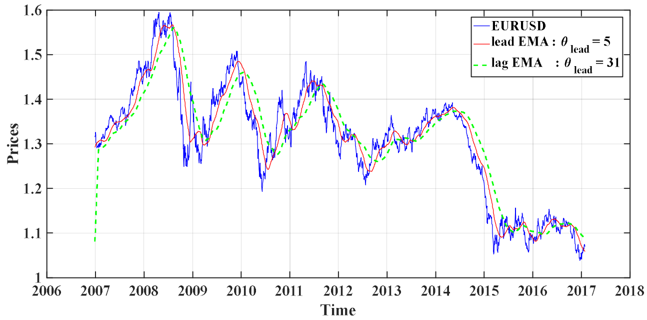

Figure 3. lead-lag EMA with $\theta_{lead} = 5$, $\theta_{lag} = 31$ with Sharp ratio 0.0463

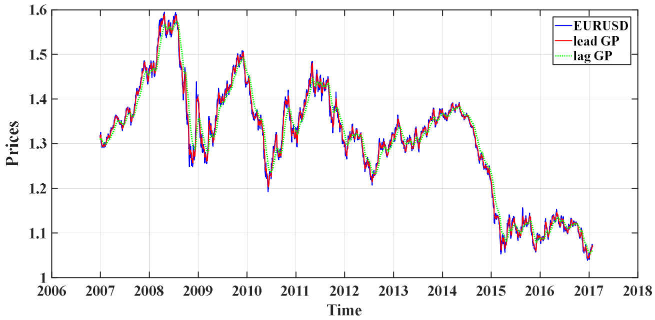

Figure 4. lead-lag Gaussian Process kernel with Sharp ratio 0.0821

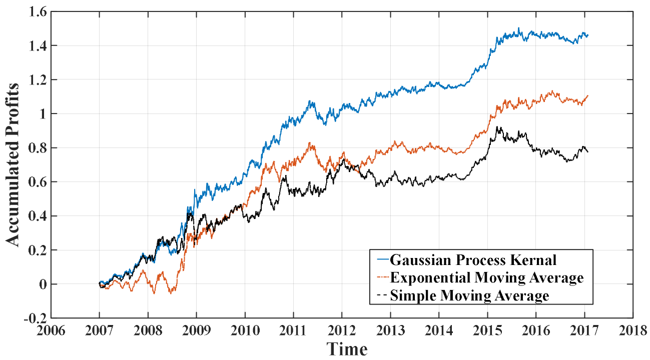

Figure 5. Performance comparison of cumulative return

VII. CONCLUSION

This paper has presented the automated system for generating of trading signal either buy or sell using two trend lines golden cross-over by Gaussian Process kernel. These trading signal are based on the concept of two lead-lag time series Gaussian Process kernel as $lead_t = GP_t(\theta_{lead})$ and $lag_t = GP_t(\theta_{lag})$. However, the solution of lead-lag Gaussian Process kernel depends on the selections of hyperparameters $\theta = \{\theta_{lead}, \theta_{lag}\}$. The objective negative Sharp ratio function $f(\theta)$ is presented to find the minimization problem for optimal values of hyperparameters $\theta = \{\theta_{lead}, \theta_{lag}\}$ by using Genetic algorithm.

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